Direct calculation of entrainment and detrainment in large eddy simulations of boundary layer clouds.

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The problem: how do we represent this:
Using this?

Siebesma, 1998

a)
Entrainment and detrainment in shallow clouds is typically calculated as a residual in the tracer budget using LES.

- Romps (2010): Time-average the entrainment fluxes over the time needed for an entire grid cell flip from cloud to environment
- Dawe and Austin (2010): Use spatial interpolation to improve the all or nothing estimate of the cloud volume.
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A more direct calculation using the relative velocity into/out of a cloud is difficult on the discrete LES grid, because the cloud surface moves at either 0 or $\frac{\Delta x}{\Delta t} \approx 15\text{–}30 \text{ m s}^{-1}$. 

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Bulk entrainment – definitions

- Define an averaging operator:

\[
\phi(z) = \frac{1}{A} \int_0^{L_x} \int_0^{L_y} \phi(x, y, z) \, dx \, dy
\]

where \( A = L_x L_y \)
Bulk entrainment – definitions

Define an averaging operator:

\[
\bar{\phi}(z) = \frac{1}{A} \int_0^{L_x} \int_0^{L_y} \phi(x, y, z) \, dx \, dy
\]

where \( A = L_x L_y \)

Separate the domain into environment and cloud core:

\[
\bar{\phi}_c = \phi_c = \frac{1}{A_c} \int \int_{\text{cloud}} \phi(x, y, z) \, dx \, dy
\]

\[
\bar{\phi}_e = \phi_e = \frac{1}{A_e} \int \int_{\text{env}} \phi(x, y, z) \, dx \, dy
\]

\[
a_c = A_c / A \text{ (fractional cloud cover)}
\]
Bulk entrainment, cont.

- The entrainment and detrainment rates are:

\[
E = -\frac{1}{A} \oint_{\hat{n} \cdot (\mathbf{u} - \mathbf{u}_i) < 0} \rho \hat{n} \cdot (\mathbf{u} - \mathbf{u}_i) \, dl
\]

\[
D = \frac{1}{A} \oint_{\hat{n} \cdot (\mathbf{u} - \mathbf{u}_i) > 0} \rho \hat{n} \cdot (\mathbf{u} - \mathbf{u}_i) \, dl
\]
Bulk entrainment, cont.

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\]

- The bulk plume/environment approximation:

\[
E_{\phi e} = -\frac{1}{A} \oint_{\hat{n} \cdot (u - u_i) < 0} \rho \hat{n} \cdot (u - u_i) \phi dl \\
D_{\phi c} = \frac{1}{A} \oint_{\hat{n} \cdot (u - u_i) > 0} \rho \hat{n} \cdot (u - u_i) \phi dl
\]

To proceed, assume \( E_{\phi} = E \) and \( D_{\phi} = D \), but ...
Clouds are surrounded by a cool moist shell
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- So entrained/detrained air properties differ from $\phi_e$, $\phi_c$
Calculating bulk $E_\phi$ and $D_\phi$

- Following Siebesma and Cuijpers, mass and tracer continuity yield:

\[
E_\phi(\phi_c - \phi_e) = -M_c \frac{\partial \phi_c}{\partial z} - \frac{\partial \rho a w' \phi' c}{\partial z} - \rho a \frac{\partial \phi_c}{\partial t} + a \rho \left( \frac{\partial \bar{\phi}}{\partial t} \right) \text{forcing}
\]
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and

$$D_\phi(\phi_c - \phi_e) = -M_c \frac{\partial \phi_e}{\partial z} + \frac{\partial \rho (1 - a) w' \phi'^e}{\partial z}$$

$$+ \rho (1 - a) \frac{\partial \phi_e}{\partial t} - \rho (1 - a) \left( \frac{\partial \bar{\phi}}{\partial t} \right) \text{forcing}$$
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- where LES is used for the rhs terms. So how do $E_\phi$ and $D_\phi$ compare to $E$ and $D$?
Romps time-averaged direct calculation for $E_d$ and $D_d$

- Romps (JAS, 2010) defines the *activity* $A$, which is 1 in a cloud core gridcell and 0 otherwise.
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- Mass continuity relates $A$ to the local entrainment and detrainment rates.

$$e(x, y, z) - d(x, y, z) = \frac{\partial}{\partial t}(A\rho) + \nabla \cdot (\rho u A)$$

- To get the direct $E_d(z)$ and $D_d(z)$:
  - Average $e - d$ over the time required for a grid cell to change state.
  - Label positive $e - d$ as $e$, negative $e - d$ as $d$.
  - Sum $e$ and $d$ over $(x, y)$ to get $E_d(z)$ and $D_d(z)$.

- Romp's result: $E_d$ and $D_d$ are about a factor of 2 larger than $E_\phi$ and $D_\phi$, and depend on the tracer type.
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Direct calculations using cloud surface interpolation

Start with the net entrainment and derive:

\[
E - D = \int_{C} \rho (u_i - u) \cdot dC
\]
\[
E - D = \rho \frac{dV}{dt} + \int_{W} \rho u \cdot dW
\]

\[
E = \max \left(0, \rho \frac{dV}{dt} + \int_{W} \rho u \cdot dW\right)
\]
\[
D = \max \left(0, -\rho \frac{dV}{dt} - \int_{W} \rho u \cdot dW\right)
\]

▶ Dawe and Austin (MWR, 2010)
Use tetrahedral interpolation to get the subgrid volume.

Where the core-environment boundary is determined by interpolating $q_v$, $q_s$, $T_v$ and $w$. 

Direct entrainment: tetrahedral
Good agreement between Romps and tetrahedral

$E_d, D_d$

$\epsilon_d, \delta_d$

but there's some dependence on numerics (note advection scheme differences)
The tetrahedral technique requires high resolution...
Because the interpolation biases single gridcell cloud area

Direct entrainment: tetrahedral
Tetrahedral fluxes can be used for $E_d/D_d$ snapshots.

a) Tetrahedral Entrainment

- 5 minute average
- 1 minute
- 1.5 seconds

b) Tetrahedral Detrainment

c) Romps Entrainment

d) Romps Detrainment

Direct entrainment: tetrahedral
$E_d/D_d$ spatial variability, 1 minute average

Tetrahedral

Romps

Direct entrainment: tetrahedral
Linking direct and bulk entrainment rates

Core: \( w > 0, \Delta T_v > 0, q_I > 0 \)
Edge: outermost core gridcell
Shell: Innermost environment gridcell
converting $E_d$ to $E_\phi$

Define shell and edge tracer values $q_{shell}$ and $q_{edge}$. These values will differ from $q_c$ and $q_e$, the mean cloud core and environment vapor values.

Can show that:

$$E_\phi = E_d - E_d \frac{q_{shell} - q_e}{q_c - q_e}$$

$$-D_d \frac{q_c - q_{edge}}{q_c - q_e}$$

Alternatively, use conditional averages to include Reynolds correlations

$$q_E = (E_\phi)_d / E_d$$

$$q_D = (D_\phi)_d / D_d$$
Corrected fluxes restore bulk tracer profile

The $q_E / q_D$ underestimate of $E$ and $D$ arises from differencing two large quantities: $q_c(E_d - D_d)$ and $(Eq)_d - (Dq)_d$. 

Linking direct and bulk entrainment
When we modify the entrainment calculation to account for negative and positive $w$, we find $w_E$, the Reynold’s correlated mean entrained vertical velocity $> 0$ and nearly as large as $w_D$. 
Why is the cloud entraining positive $w$?

Updrafts produce newly buoyant parcels well above cloudbase.
Two new techniques to directly calculate entrainment show entrainment rates roughly two times higher than bulk values.
Summary

- Two new techniques to directly calculate entrainment show entrainment rates roughly two times higher than bulk values.
- Much of the difference between bulk and direct calculations is due to the influence of the heterogeneous cloud shell/edge, but Reynolds correlations also play a significant role, particularly for momentum (see our JAS submission).
Two new techniques to directly calculate entrainment show entrainment rates roughly two times higher than bulk values. Much of the difference between bulk and direct calculations is due to the influence of the heterogeneous cloud shell/edge, but Reynolds correlations also play a significant role, particularly for momentum (see our JAS submission). Tetrahedral interpolation can be applied to individual clouds, and rapidly changing boundary layers, either over a cloud life cycle or in a snapshot.
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Much of the difference between bulk and direct calculations is due to the influence of the heterogeneous cloud shell/edge, but Reynolds correlations also play a significant role, particularly for momentum (see our JAS submission).

Tetrahedral interpolation can be applied to individual clouds, and rapidly changing boundary layers, either over a cloud life cycle or in a snapshot.

The interpolation technique is applicable in general to any flow through a material surface in a three-dimensional model.
Thanks to:

- Marat for providing SAM
- Support from the Canadian Foundation for Climate and Atmospheric Science
Romps - tetrahedral correlation

a) $\epsilon$

- Black line: upwind
- Gray line: MPDATA

b) $\delta$

95%
static energy
ARM diurnal

(a) \( \frac{q_E - q_{te}}{q_{te} - q_{te}} \)

(b) \( \frac{q_{te} - q_D}{q_{te} - q_{te}} \)