

Direct calculation of entrainment and  
detrainment in large eddy simulations of  
boundary layer clouds.

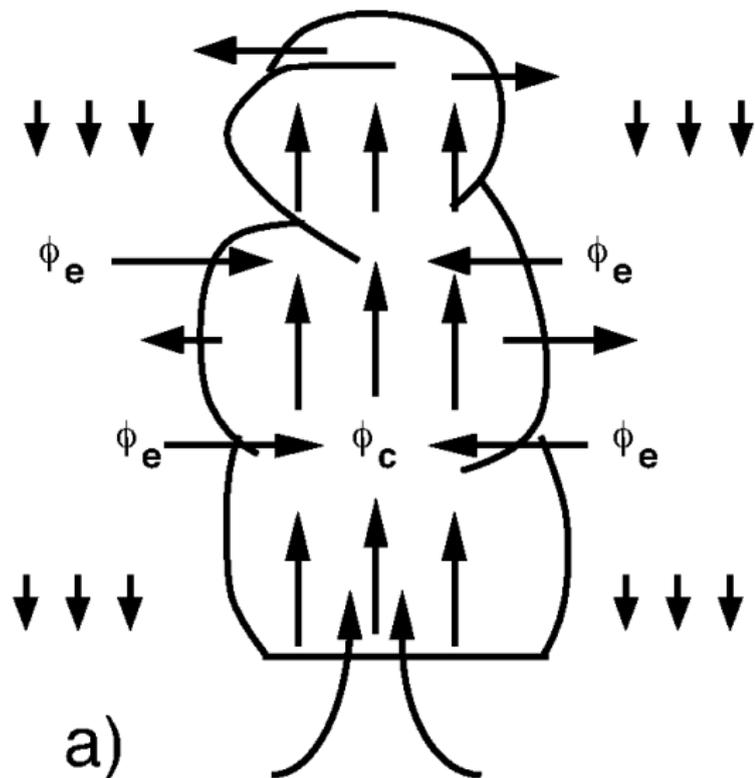
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University of British Columbia

September, 2010

The problem: how do we represent this:



Using this?



Siebesma, 1998

# Introduction

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  - ▶ Romps (2010): Time-average the entrainment fluxes over the time needed for an entire grid cell flip from cloud to environment
  - ▶ Dawe and Austin (2010): Use spatial interpolation to improve the all or nothing estimate of the cloud volume.

## Bulk entrainment – definitions

- ▶ Define an averaging operator:

$$\overline{\phi(z)} = \frac{1}{A} \int_0^{L_x} \int_0^{L_y} \phi(x, y, z) dx dy$$

where  $A = L_x L_y$

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- ▶ Separate the domain into environment and cloud core:

$$\overline{\phi_c} = \phi_c = \frac{1}{A_c} \int \int_{\text{cloud}} \phi(x, y, z) dx dy$$
$$\overline{\phi_e} = \phi_e = \frac{1}{A_e} \int \int_{\text{env}} \phi(x, y, z) dx dy$$
$$a_c = A_c/A \text{ (fractional cloud cover)}$$

## Bulk entrainment, cont.

- ▶ The entrainment and detrainment rates are:

$$E = -\frac{1}{A} \oint_{\hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) < 0} \rho \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) dl$$

$$D = \frac{1}{A} \oint_{\hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) > 0} \rho \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) dl$$

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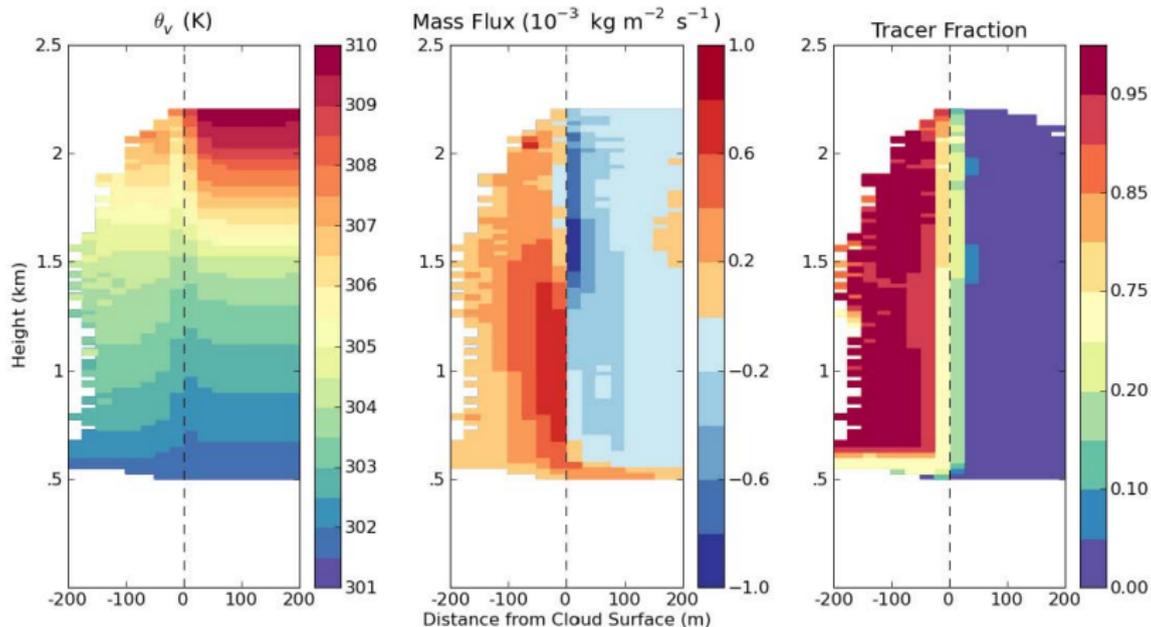
- ▶ The bulk plume/environment approximation:

$$E_\phi \phi_e = -\frac{1}{A} \oint_{\hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) < 0} \rho \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) \phi dl$$

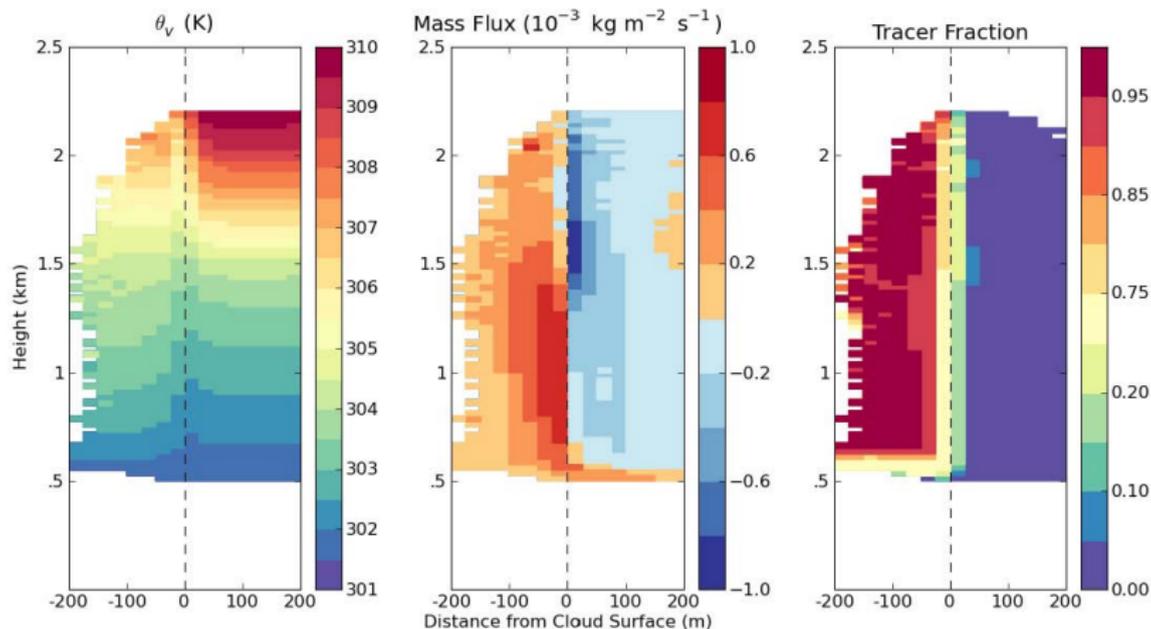
$$D_\phi \phi_c = \frac{1}{A} \oint_{\hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) > 0} \rho \hat{\mathbf{n}} \cdot (\mathbf{u} - \mathbf{u}_i) \phi dl$$

to proceed, assume  $E_\phi = E$  and  $D_\phi = D$ , but ...

# Clouds are surrounded by a cool moist shell



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- ▶ So entrained/detrained air properties differ from  $\phi_e$ ,  $\phi_c$

## Calculating bulk $E_\phi$ and $D_\phi$

- ▶ Following Siebesma and Cuijpers, mass and tracer continuity yield:

$$E_\phi(\phi_c - \phi_e) = -M_c \frac{\partial \phi_c}{\partial z} - \frac{\partial \rho a \overline{w' \phi'^c}}{\partial z} - \rho a \frac{\partial \phi_c}{\partial t} + a \rho \left( \frac{\partial \bar{\phi}}{\partial t} \right)_{\text{forcing}}$$

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- ▶ where LES is used for the rhs terms. So how do  $E_\phi$  and  $D_\phi$  compare to  $E$  and  $D$ ?

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- ▶ To get the direct  $E_d(z)$  and  $D_d(z)$ :
  - ▶ Average  $e - d$  over the time required for a grid cell to change state
  - ▶ Label positive  $e - d$  as  $e$ , negative  $e - d$  as  $d$
  - ▶ Sum  $e$  and  $d$  over  $(x, y)$  to get  $E_d(z)$  and  $D_d(z)$ .

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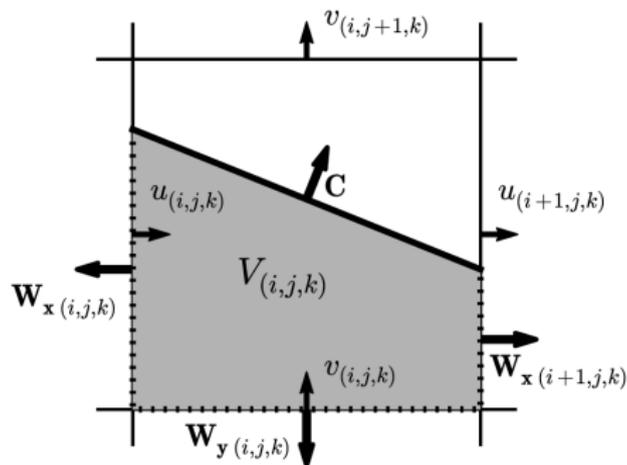
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  - ▶ Sum  $e$  and  $d$  over  $(x, y)$  to get  $E_d(z)$  and  $D_d(z)$ .
- ▶ Romp's result:  $E_d$  and  $D_d$  are about a factor of 2 larger than  $E_\phi$  and  $D_\phi$ , and depend on the tracer type

# Direct calculations using cloud surface interpolation

Start with the net entrainment and derive:



$$E - D = \int_C \rho(\mathbf{u}_i - \mathbf{u}) \cdot d\mathbf{C}$$

$$E - D = \rho \frac{dV}{dt} +$$

$$\int_W \rho \mathbf{u} \cdot d\mathbf{W}$$

$$E =$$

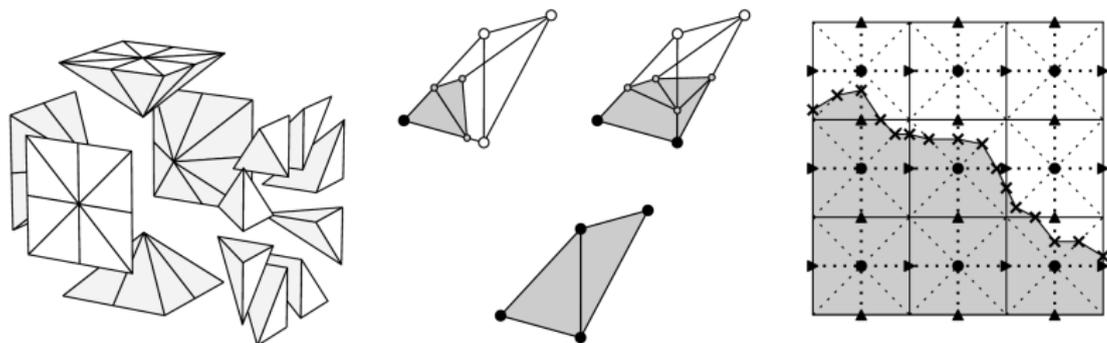
$$\max \left( 0, \rho \frac{dV}{dt} + \int_W \rho \mathbf{u} \cdot d\mathbf{W} \right)$$

$$D =$$

$$\max \left( 0, -\rho \frac{dV}{dt} - \int_W \rho \mathbf{u} \cdot d\mathbf{W} \right)$$

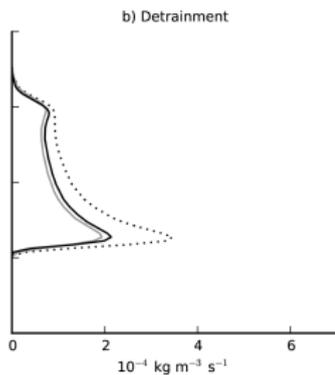
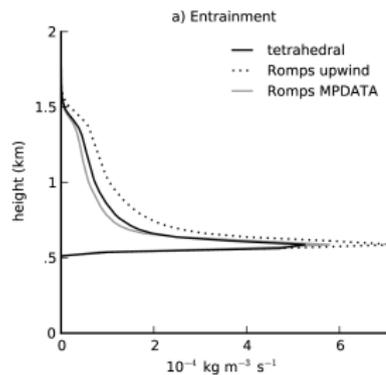
- Dawe and Austin (MWR, 2010)

# Use tetrahedral interpolation to get the subgrid volume

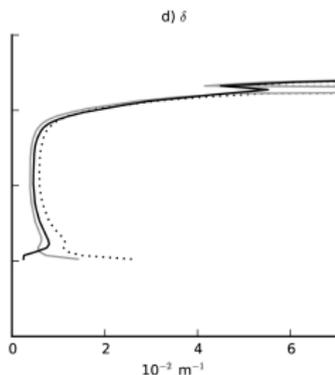
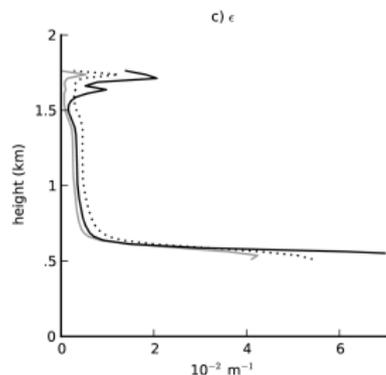


- ▶ Where the core-environment boundary is determined by interpolating  $q_V$ ,  $q_S$ ,  $T_V$  and  $w$ .

# Good agreement between Romps and tetrahedral



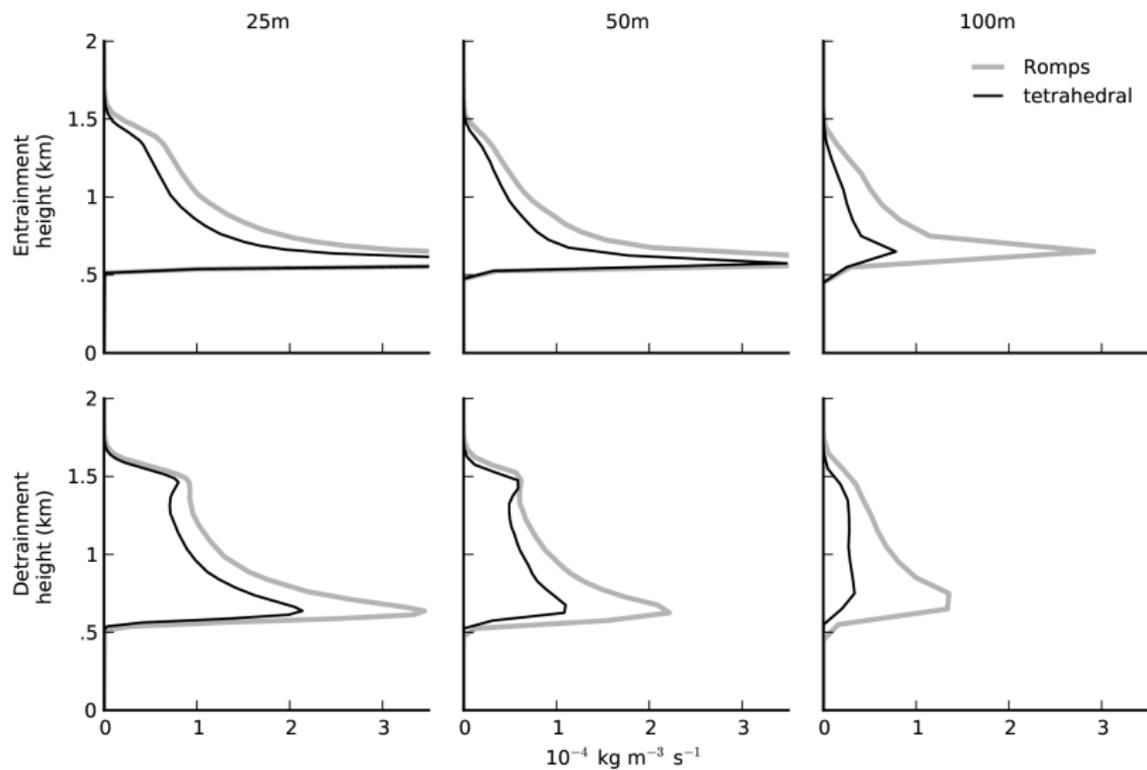
$$E_d, D_d$$



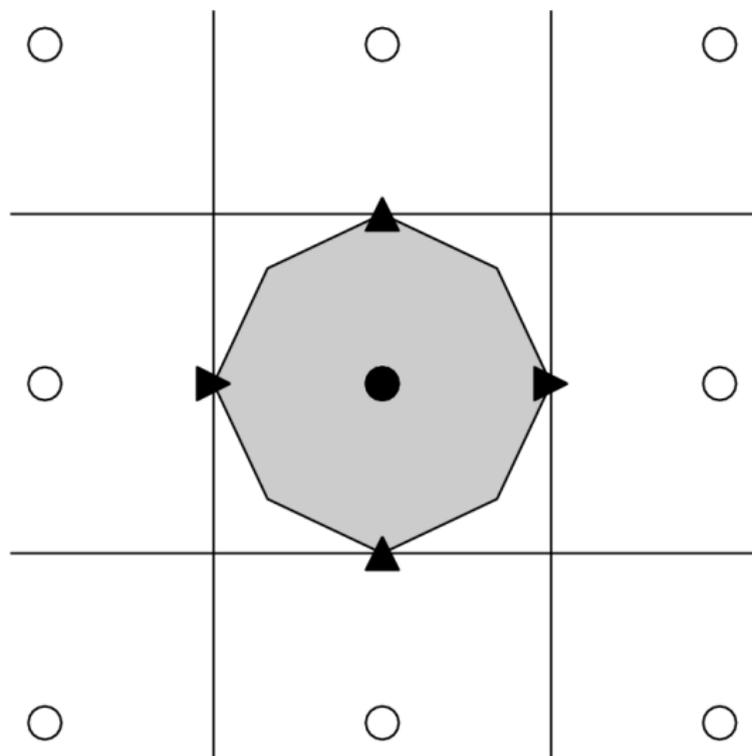
$$\epsilon_d, \delta_d$$

but there's some dependence on numerics (note advection scheme differences)

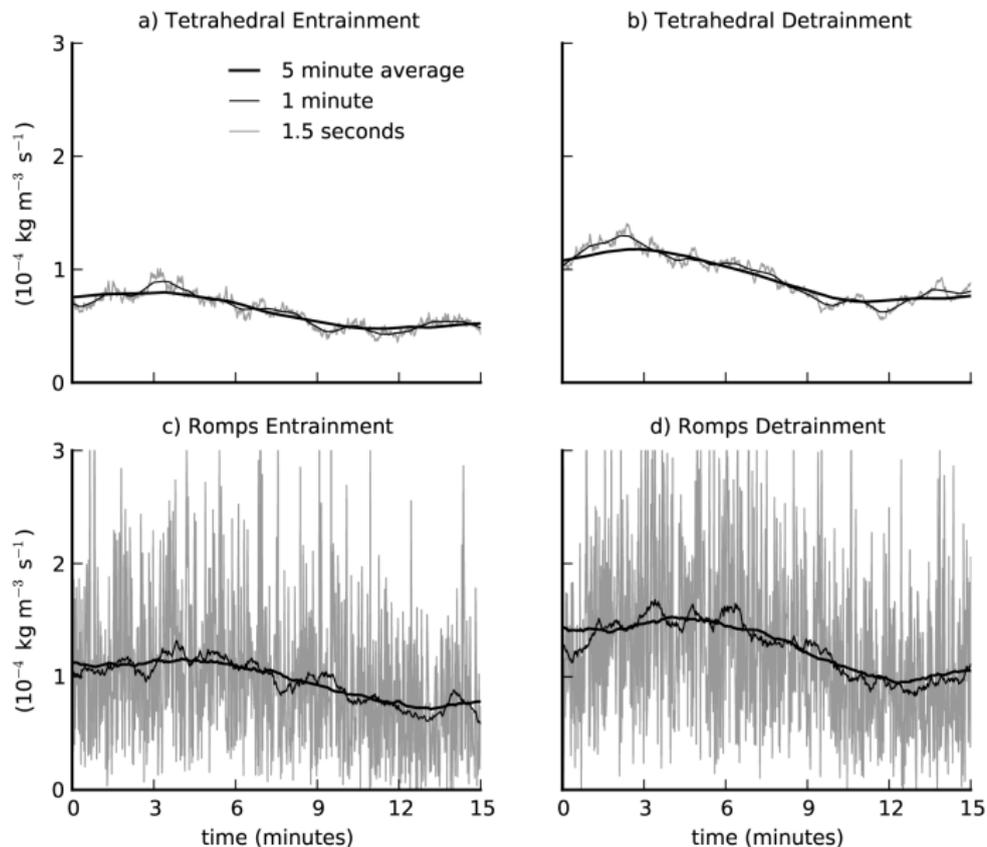
# The tetrahedral technique requires high resolution ...



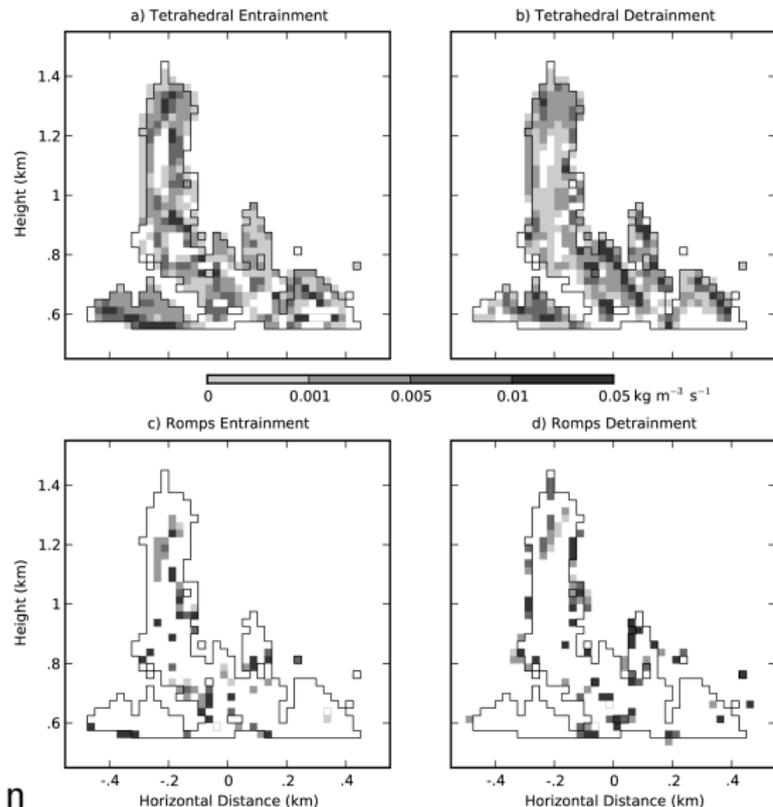
Because the interpolation biases single gridcell cloud area



# Tetrahedral fluxes can be used for $E_d/D_d$ snapshots



# $E_d/D_d$ spatial variability, 1 minute average

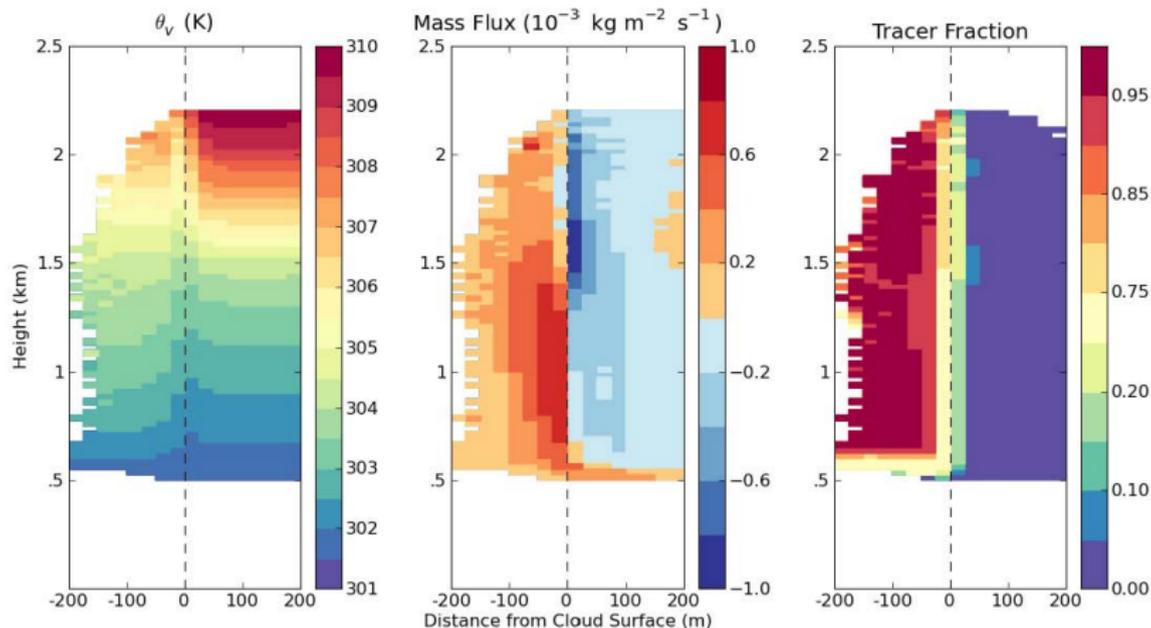


Tetrahedral

Roms

n

# Linking direct and bulk entrainment rates

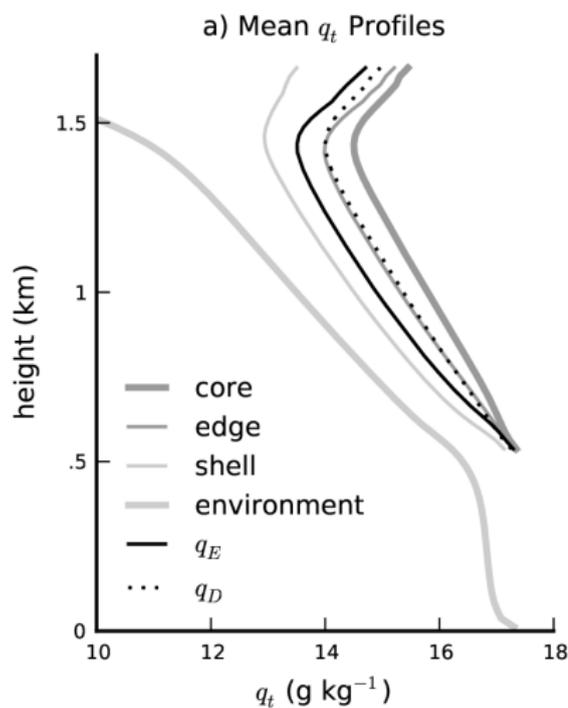


Core:  $w > 0$ ,  $\Delta T_v > 0$ ,  $q_l > 0$

Edge: outermost core gridcell

Shell: Innermost environment gridcell

## converting $E_d$ to $E_\phi$



Define shell and edge tracer values  $q_{shell}$  and  $q_{edge}$ .

These values will differ from  $q_c$  and  $q_e$ , the mean cloud core and environment vapor values

Can show that:

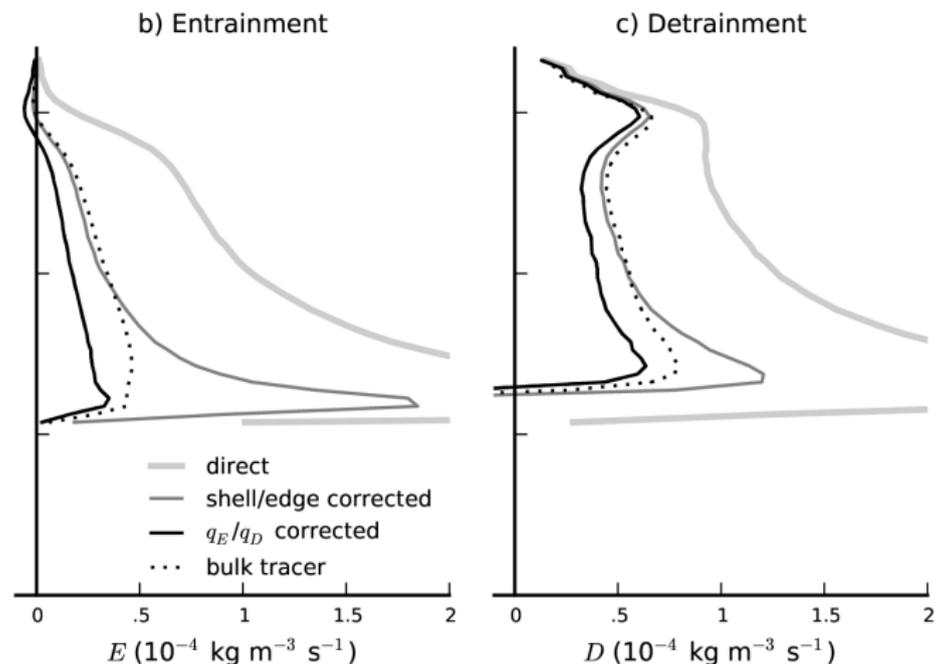
$$E_\phi = E_d - E_d \frac{q_{shell} - q_e}{q_c - q_e} - D_d \frac{q_c - q_{edge}}{q_c - q_e}$$

Alternatively, use conditional averages to include Reynolds correlations

$$q_E = (E\phi)_d / E_d$$

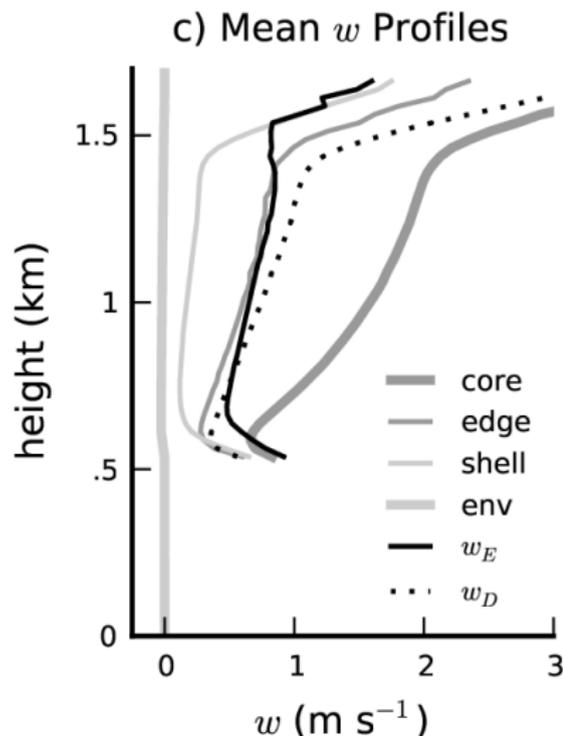
$$q_D = (D\phi)_d / D_d$$

# Corrected fluxes restore bulk tracer profile



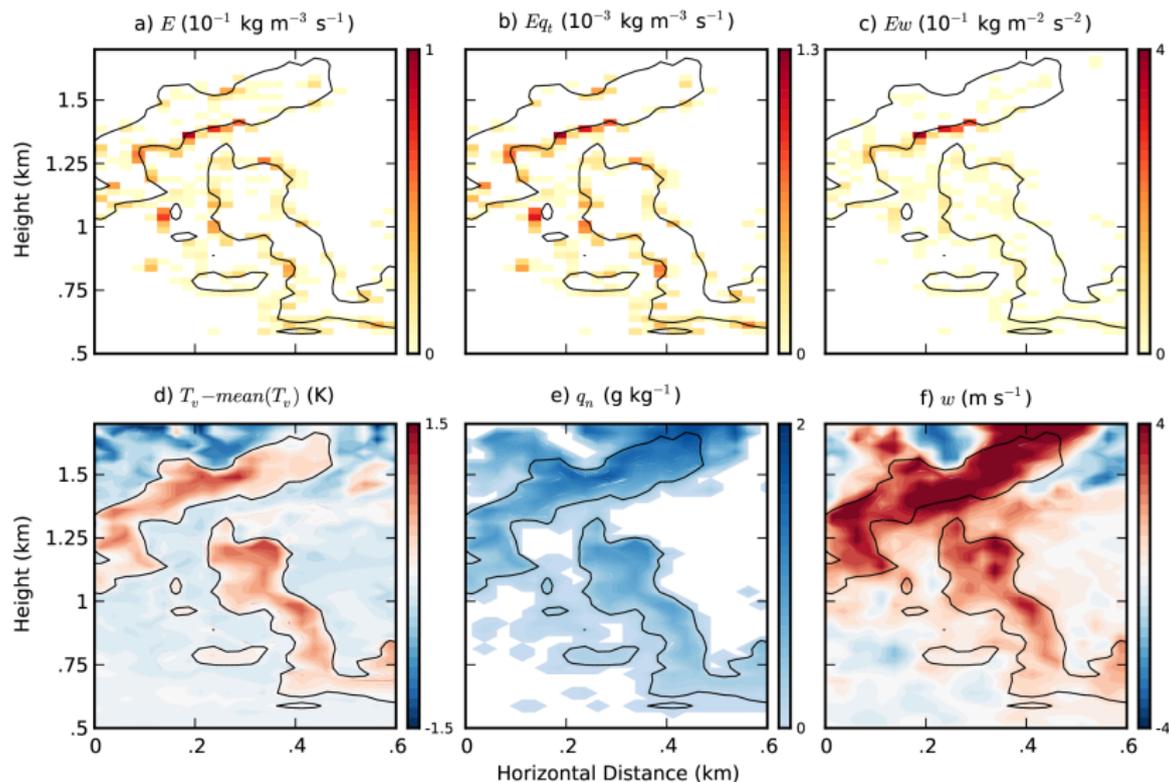
The  $q_E / q_D$  underestimate of  $E$  and  $D$  arises from differencing two large quantities:  $q_c(E_d - D_d)$  and  $(Eq)_d - (Dq)_d$ .

## Vertical momentum



When we modify the entrainment calculation to account for negative and positive  $w$ , we find  $w_E$ , the Reynolds's correlated mean entrained vertical velocity  $> 0$  and nearly as large as  $w_D$ .

# Why is the cloud entraining positive $w$ ?



Updrafts produce newly buoyant parcels well above cloudbase.

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- ▶ Tetrahedral interpolation can be applied to individual clouds, and rapidly changing boundary layers, either over a cloud life cycle or in a snapshot.

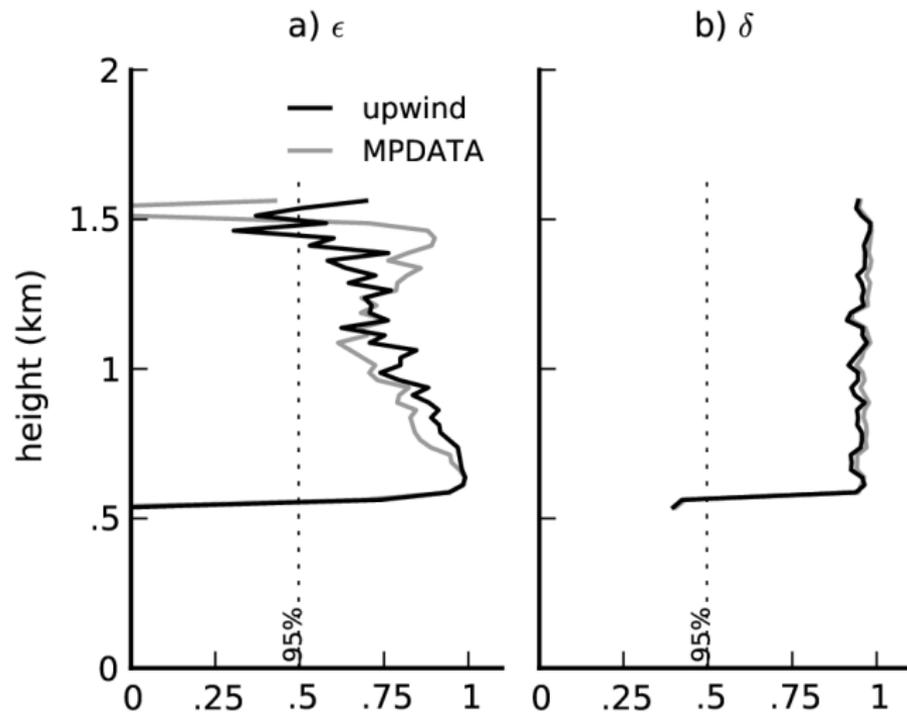
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- ▶ Tetrahedral interpolation can be applied to individual clouds, and rapidly changing boundary layers, either over a cloud life cycle or in a snapshot.
- ▶ The interpolation technique is applicable in general to any flow through a material surface in a three-dimensional model.

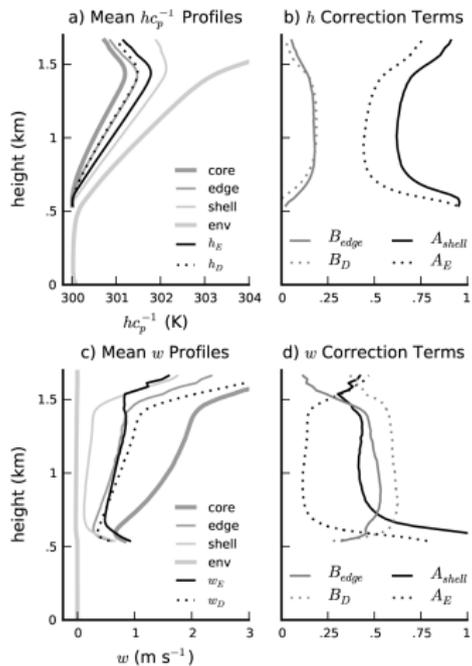
## Thanks to:

- ▶ Marat for providing SAM
- ▶ Support from the Canadian Foundation for Climate and Atmospheric Science

# Romps - tetrahedral correlation



# static energy



# ARM diurnal

